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THE EFFECT OF SPANWISE WALL TEMPERATURE GRADIENTS ON NOMINALLY --ETC(U)  
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THE EFFECT OF SPANWISE WALL TEMPERATURE GRADIENTS ON NOMINALLY TWO-DIMENSIONAL  
LAMINAR BOUNDARY LAYERS

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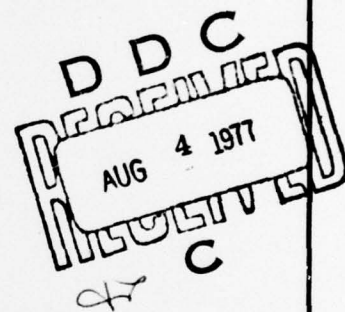
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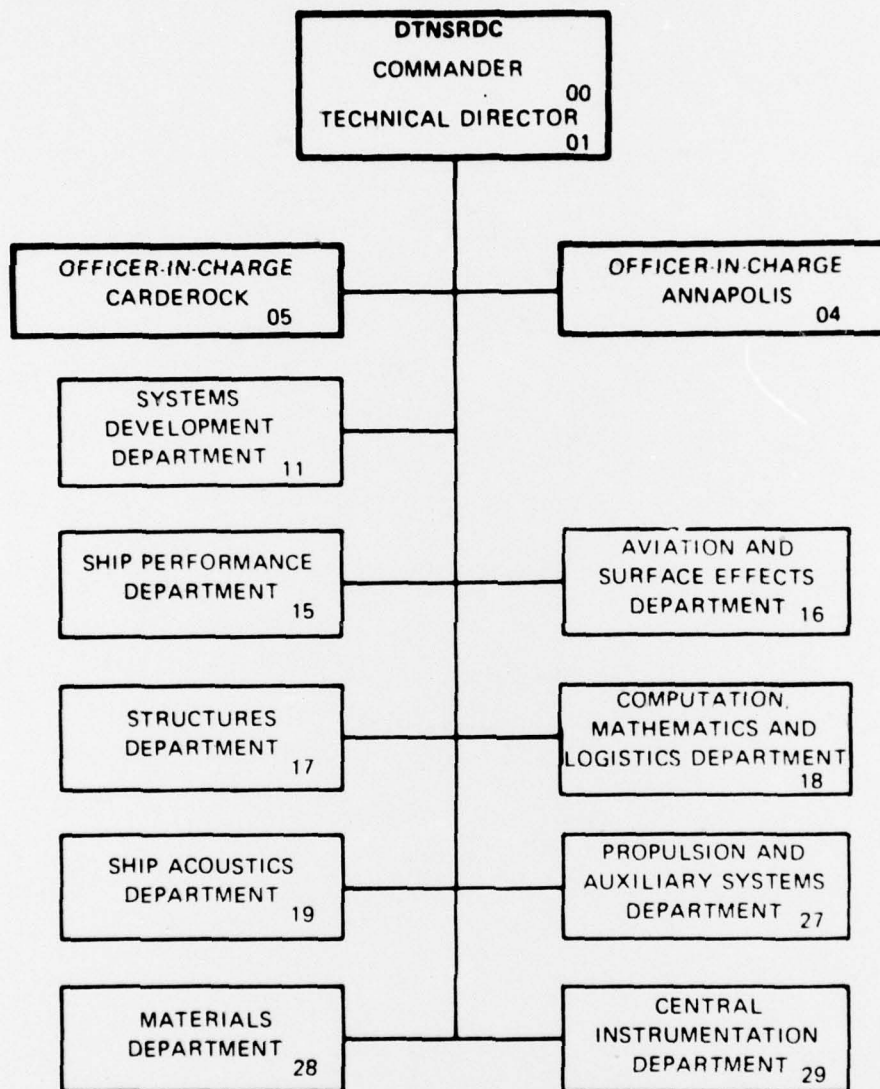


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SPD-783-01

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### ABSTRACT

It is argued that spanwise temperature gradients in wall heating do not affect the boundary layer crossflow to first order if the length scale of the spanwise temperature variation is much greater than the boundary layer thickness. Implications for linear instability theory and transition are discussed for flow about two-dimensional or axisymmetric bodies.

### ADMINISTRATIVE INFORMATION

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### INTRODUCTION

Heating a body surface is a means by which a laminar water boundary layer can be stabilized and transition to turbulence delayed. Such techniques for boundary layer stabilization have been considered for the planar or axisymmetric flow over bodies. If the heating of the wall boundary is uniform in the spanwise or circumferential directions, then the boundary layer remains planar or axisymmetric respectively. Two-dimensional linear instability theory has been commonly used to correlate natural transition in planar and axisymmetric boundary layers<sup>1</sup>.

A question which then arises is the effect on transition of spanwise or circumferential wall temperature variations. The planar case will be discussed below. The conclusions, which are qualitative, also hold for the axisymmetric case.

There is little hope of understanding the effect of spanwise temperature gradients on transition since at present, very little is understood about transition even in perfectly two-dimensional flows with uniform boundary conditions. However, qualitative effects on natural transition of such factors as pressure gradient, wall heating, and suction in perfectly planar flows have proved to be predictable by linear instability theory. Thus, a discussion of the effects of spanwise wall

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<sup>1</sup>Mack, L.M., "Transition and Laminar Instability," JPL Publication 17-15, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA (1977).

temperature variations on linear instability theory may prove useful in assessing the effect on transition.

There are two ways in which spanwise variations in wall temperature can affect boundary layer instability. A spanwise temperature variation might induce crossflows in the boundary layer. The resulting three-dimensional boundary layer might be considerably less stable than the nominally two-dimensional boundary layer with the same streamwise temperature variation. Even if the boundary layer remains planar, the growth of disturbances might be affected directly by the temperature gradients. We shall argue below that neither of these possibilities are important factors in linear instability theory if the spanwise scale of variation of the surface temperature is much greater than the boundary layer thickness.

#### DIRECT ACTION OF SPANWISE TEMPERATURE VARIATIONS ON THE LAMINAR BOUNDARY LAYER

Consider the case of a flat plate of infinite span past which a stream flows in a direction perpendicular to the leading edge and parallel to the plane of the plate. Let the  $(x, y, z)$  cartesian reference frame be placed on the plate with its origin in the leading edge, its  $x$ -axis pointing downstream, its  $y$ -axis spanwise and its  $z$ -axis normal to the plate. An  $x$ -wise pressure gradient  $dp/dx$  which is uniform spanwise can be assumed imposed by some external source. Let  $(u, v, w)$  be the boundary layer velocity components in the  $(x, y, z)$  directions respectively and let  $T$  be the temperature in the boundary layer. Based on the data given by Schlichting<sup>2</sup> on the variation of water properties with temperature, one can conclude that the density  $\rho$  and thermal diffusivity,  $\alpha$  can be taken as constant whereas the kinematic viscosity  $\nu$  is a function of temperature  $T$  only.

The heated water three-dimensional boundary layer equations can then be written as (see reference (2)).

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<sup>2</sup>Schlichting, H., "Boundary Layer Theory," 6th ed., McGraw-Hill Book Co., New York (1968).

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad [1a]$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{dp}{dx} + \frac{\partial}{\partial z} \left( \nu \frac{\partial u}{\partial z} \right) \quad [1b]$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = \frac{\partial}{\partial z} \left( \nu \frac{\partial v}{\partial z} \right) \quad [1c]$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = m \frac{\partial^2 T}{\partial z^2} \quad [1d]$$

for continuity, streamwise and spanwise momentum and temperature, respectively;  $m$  in equation [1d] is  $a/\nu_\infty$  where  $\nu_\infty$  is the kinematic viscosity at the constant temperature  $T_\infty$  at large distances from the boundary layer. The boundary conditions are:

$$u = v = w = 0 \quad \text{at } z = 0 \quad [2a]$$

$$u = U(x), \quad v = 0 \quad \text{at } z = \delta \quad [2b]$$

$$T = T_w(x, y) \quad \text{at } z = 0 \quad [2c]$$



$$T = T_{\infty} \quad \text{at} \quad z = \delta$$

[2d]

where  $U(x)$  is the inviscid surface velocity past the plate and  $\delta$  is the boundary layer thickness.

It is a trivial matter to see by inspection of [1] and [2] that a solution of [1] and [2] satisfying the initial condition  $v(x, y, z) = 0$  at  $x = 0$  is  $v = 0$  and  $u$ ,  $w$ , and  $T$  are solutions of the resulting two-dimensional equations. Assuming the uniqueness of solutions of the boundary layer equations, this implies that within first-order boundary layer theory, spanwise temperature variations cannot cause a three-dimensional flow. Note that the requirement that the scale of variation of  $T_w(x, y)$  in the  $y$  direction must be much greater than the boundary layer thickness is necessary so that the spanwise temperature diffusion term  $m \partial^2 T / \partial y^2$  can be neglected in equation [1d].

The above argument does not imply that there is no crossflow  $v$  generated by spanwise temperature gradients. It only says that such crossflows are too weak to be detected by first-order boundary layer theory. The induced crossflow should show up in second-order boundary layer theory because examination of equations [1] show that the coupling of the spanwise temperature gradients to the crossflow momentum equation is indirect through the continuity equation (i.e., there is no term in equation [1c] which is independent of  $v$ ). For instance, the solution of [1] under boundary conditions (2) yields planar boundary layers  $u(x, z; y)$ ,  $w(x, z; y)$  and  $T(x, z; y)$  which are different for each value of  $y$ . That is, these planar boundary layers are parameterized in  $y$ . As a consequence, the boundary layer thickness  $\delta$  is also a function of  $y$ . In second-order boundary layer theory, the boundary layer thickness perturbs the external inviscid flow. With spanwise temperature gradients, the boundary layer perturbation of the external inviscid flow is a function of  $y$ . Thus, the second-order pressure gradient imposed on the second-order boundary layer flow has a component in the  $y$ -direction and hence,

drives the crossflow  $v$ . A similar situation occurs for the nominally two-dimensional external flow along the generators of a cylinder of noncircular cross section.<sup>3</sup> The analysis of reference (3) can easily be modified to apply to the case of crossflow induced by spanwise temperature gradients.

The main point of this discussion is that the order of magnitude of the cross flow velocity  $v$  induced by the spanwise pressure gradient is only of order  $U(x)/R_x^{1/2}$  where  $R_x = U(x)x/\nu$ . Because of the large Prandtl number of water, where Prandtl number  $P_r \equiv \nu/a$ , the temperature effects occur in a thin boundary layer imbedded in the velocity boundary layer. The ratio of the thickness of the temperature boundary layer to velocity boundary layer is of the order of  $1/P_r$ . Thus, one can expect the order of magnitude of the cross flow  $v$  induced by spanwise temperature variations actually to be of the order of  $U(x)/(\sqrt{R_x} P_r)$ . The Prandtl number  $P_r \approx 6$  for water at temperatures near 15 deg C.

#### THE EFFECTS OF SPANWISE TEMPERATURE GRADIENTS ON LINEAR INSTABILITY THEORY

The conventional linear instability theory for heated water boundary layers subject to the parallel flow approximation is based on the eigenvalue problem

$$\begin{aligned} & \mu (D^4 \varphi - 2 \tilde{\alpha}^2 D^2 \varphi + \tilde{\alpha}^4 \varphi) + 2 (D \mu) D (D^2 \varphi - \tilde{\alpha}^2 \varphi) \\ & + (D^2 \mu) (D^2 \varphi + \tilde{\alpha}^2 \varphi) - i R (\alpha u + \beta v - \omega) (D^2 \varphi \\ & - \tilde{\alpha}^2 \varphi) + i R (\alpha D^2 u + \beta D^2 v) \varphi = 0 \end{aligned} \quad [3a]$$

with boundary conditions

$$\varphi = D \varphi = 0 \quad \text{at } z = 0 \quad [3b]$$

<sup>3</sup>Rubin, S.G. and Mummalo, F.J., "Boundary-Layer-Induced Potential Flow on an Elliptic Cylinder", J. Fluid Mech., 66, pt. 1 (1974).

$$\phi \rightarrow 0 \quad \text{at } z \rightarrow \infty$$

[3c]

where  $D \equiv \frac{d}{dz}$  and  $R$  is the boundary layer Reynolds number. The linear instability problem (3) describes the propagation of plane waves in a boundary layer with mean streamwise velocity  $u$  and crossflow velocity  $v$ . The wave front has a normal in the direction  $\hat{n} = \alpha_r \hat{i} + \beta_r \hat{j}$  where  $\hat{i}$  and  $\hat{j}$  are unit tangent vectors to the  $x$  and  $y$  axes, respectively. The wave numbers  $\alpha_r$  and  $\beta_r$  are the real parts of  $\alpha$  and  $\beta$ , respectively, and  $\bar{\alpha}^2 \equiv \alpha^2 + \beta^2$ . The imaginary parts of  $\alpha$  and  $\beta$ ,  $\alpha_i$  and  $\beta_i$ , respectively, determine the growth rate of the disturbance  $\bar{\alpha}_i$  which is defined by

$$\bar{\alpha}_i \equiv (\alpha_i^2 + \beta_i^2)^{1/2}$$

(see reference (1)). The viscosity ratio  $\mu(T)$  is defined by

$$\mu(T) \equiv \nu(T) / \nu_\infty$$

and represents the only direct effect of temperature variation in the boundary layer on the instability problem (3). The temperature affects problem (3) indirectly through its modification from unheated flows of the mean velocity profiles  $u$  and  $v$ .

The question of how the spanwise temperature gradients affect the disturbance growth rates can be answered based on problem (3). The first thing to note is that the direct action of temperature, through the viscosity ratio  $\mu(T)$  cannot distinguish temperature variation in the plane of the boundary layer. Hence, the only direct spanwise temperature variation effect on the instability problems is to make  $\alpha_i$  a parametric function of  $y$ .

The second effect, of spanwise temperature variations, in the instability problem (3) is the generation of a crossflow velocity  $v$ . But we have argued that this is a higher-order boundary layer effect; hence it should not be included in (3a) unless other higher-order effects



are also included. In particular, the downstream variation in boundary layer thickness and the vertical mean velocity  $w$ , i.e., non-parallel flow effects, should also be included if the very small crossflow  $v$  is included. Non-parallel flow effects have been included in modified linear instability studies in reference (4). Their effects are small for mean flows with a favorable streamwise pressure gradient, but their effects on growth rates can be substantial in unfavorable streamwise pressure gradients. Thus, the true effects, on the growth rates of disturbances, of small crossflows  $v$  cannot be properly assessed outside the scope of non-parallel flow instability theory. But taking a clue from non-parallel flow effects, one can surmise that the crossflow effect is small for favorable pressure gradient boundary layers.

Even in an inconsistent approximation in which very small crossflows  $v$  are included in the parallel flow approximation, one can expect a negligible effect on disturbance growth rates. This can be easily seen because  $v$  is generally of the order of magnitude  $u/\sqrt{R_x}$  and  $R_x$  is usually in the range of  $10^5$  to  $10^6$  in regions where disturbance growth is expected. Thus,  $v$  is not likely to be greater than one percent of  $u$  and thus, not likely to affect substantially the growth rate  $\bar{\alpha}_i$ . This remains true even for very large  $R_x$  because  $\bar{\alpha}$  becomes very large. Very large  $\bar{\alpha}$  corresponds to very high frequency (short wavelength) disturbances which can be shown to be very strongly damped.

The most subtle possible effect of spanwise temperature variation on the growth of small disturbances involves not the crossflow  $v$  at all, but simply the direction of propagation of the most unstable disturbance. Disturbance growth is measured by summing the growth rates of a disturbance following its propagation in the direction of its group velocity.<sup>1</sup> The spanwise variation of the viscosity ratio  $\mu$  (due to spanwise variation of the temperature) then alternates the path along which a disturbance wave packet propagates and thus could alter the total amplification that a disturbance attains. This is due to the fact that the path of a disturbance enroute to its penultimate amplitude has a length that generally scales on the body's

<sup>4</sup> Saric, W.S. and Nayfeh, A.H., "Non-Parallel Stability of Boundary Layer Flows," Phys. Fluids, 18, pp. 945-950 (1975).



scale. Thus, the disturbance could possibly experience the effects of the entire spanwise temperature variation. However, these excursions of disturbances through the variable temperature field need not cause them to necessarily attain large amplification if the temperature is sufficiently large to maintain stability even in the nominally low temperature regions.

#### SUMMARY

It has been argued that spanwise surface temperature variations are very unlikely to lead directly to development of non-negligible crossflows in the mean boundary layer. Nor will these temperature gradients have a significant influence on the growth rates of disturbances. These statements are true as long as the scale of the spanwise temperature variation is much larger than the boundary layer thickness. Thus, the only requirement that such temperature variations do not induce premature transition (or at least not premature amplification of disturbances) is that the nominally low spots of temperature be high enough that the two-dimensional boundary layer is not very unstable there.

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1. Mack, L.M., "Transition and Laminar Instability," JPL Publication 77-15, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA (1977).
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